10.1: Parametric Equations in R^2
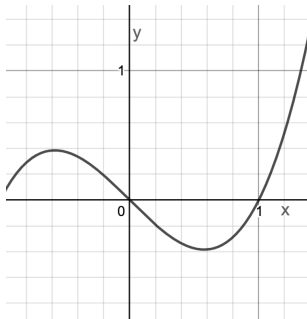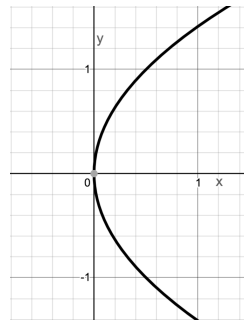
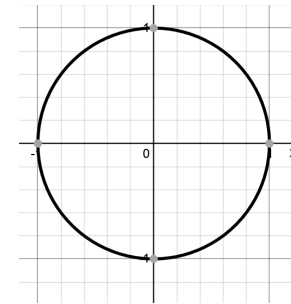*(For an interesting introduction to parametric equations, see "Quick Intuition about Parametric Equation"s on the 5B page)*

We have learned that the graphs of equations in two variables are curves in R^2 showing a relationship between the two variables, suppose x and y. Sometimes the relation can be expressed as a function, other times it cannot.

$$y = f(x) = x^3 - x \qquad\qquad x = g(y) = y^2 \qquad\qquad x^2 + y^2 = 1$$

In this section, we will consider another way to write equations for curves in R^2, called _____.
This gives us the opportunity to
   (1)_____ and

   (2) _____(later)

Showing dependence on a third variable

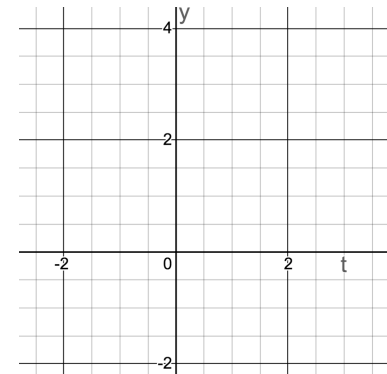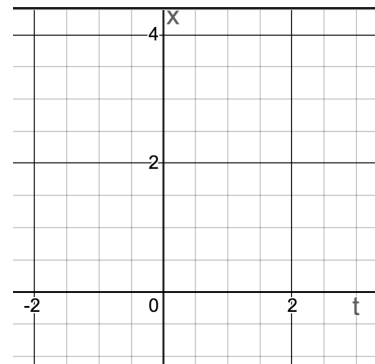$$\text{ice cream} = (\text{sunscreen})^2$$

Often, the parameter is *time*.

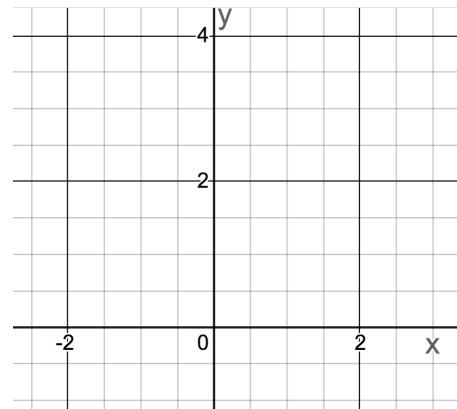Example: Sketch $\begin{cases} x = 1 - t \\ y = t^2 \end{cases}$

Methods:

1. Plot points (last resort!)

2. Consider the graphs x(t) and y(t) separately to determine horizontal and vertical behavior. Then combine these "behaviors".
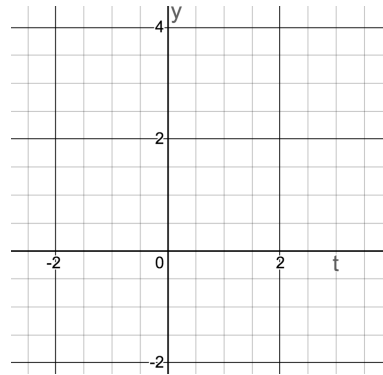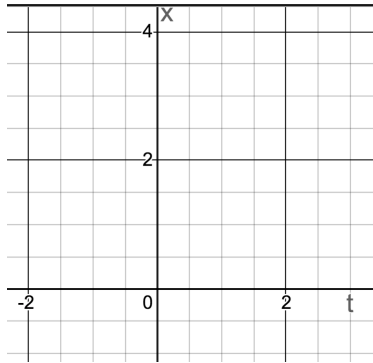
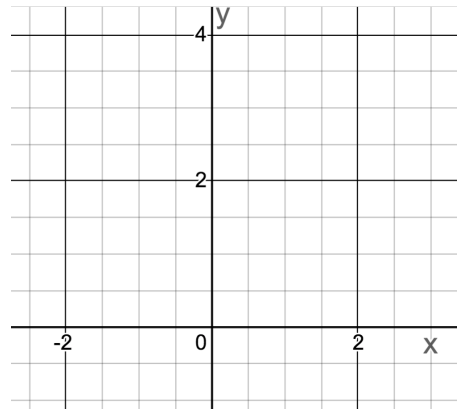3. Eliminate the parameter (best, if possible). Caution: _____

What if the problem was $\begin{cases} x = 1 - t^2 \\ y = t^4 \end{cases}$

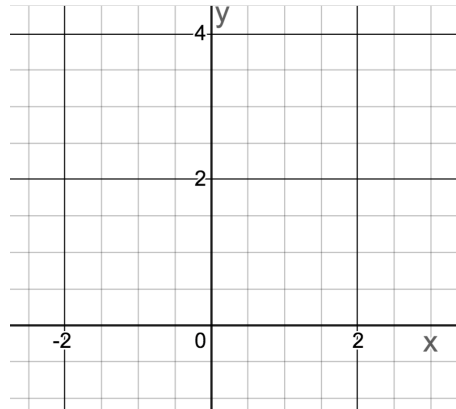*Always show direction of increasing t.*

What if the problem was $\begin{cases} x = 1 - t^2 \\ y = t^4 \end{cases}$

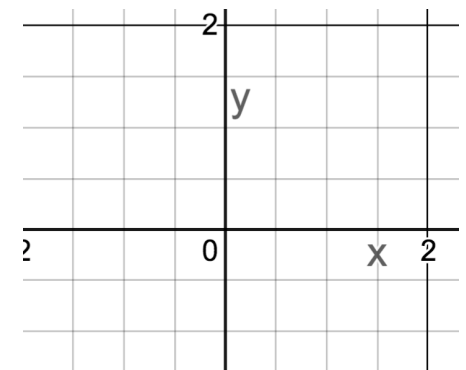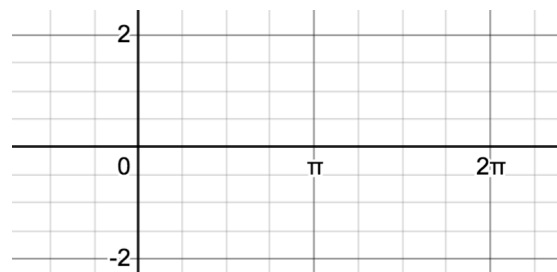1. Eliminate the parameter (best, if possible). Caution: _____
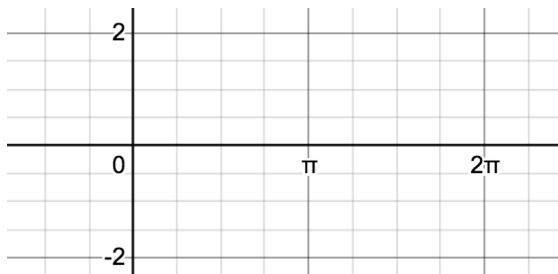
Example: Sketch $\begin{cases} x = e^t \\ y = e^{2t} \end{cases}$

Expressing an equation that is not a function in terms of two which are.

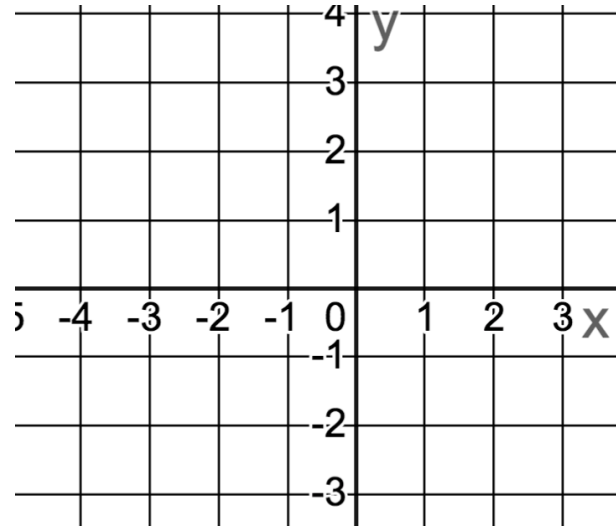Example: Sketch $\begin{cases} x = \cos t \\ y = \sin t \end{cases}$
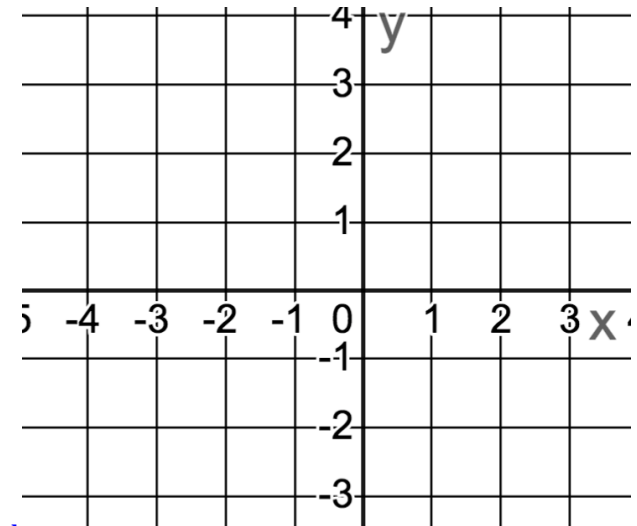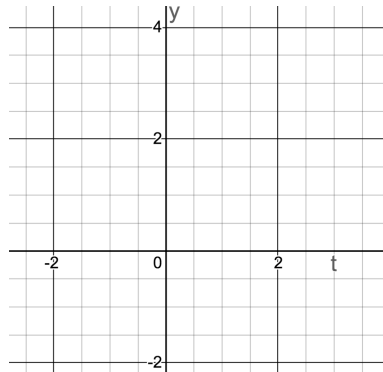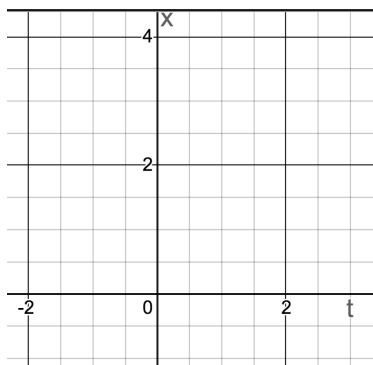
First consider x(t) and y(t) separately

animation from 5B page: https://www.desmos.com/calculator/vslkzgeocx

Sketch $\begin{cases} x = 2\sin t \\ y = 3\cos t \end{cases}$
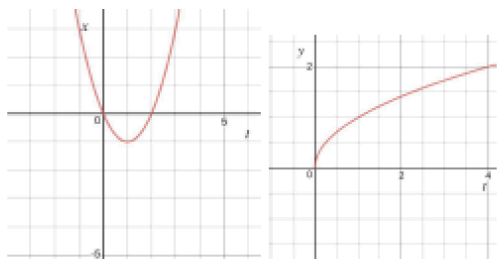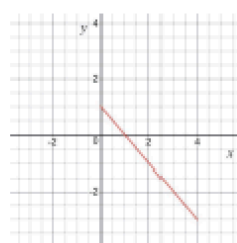

Consider x(t), y(t) <u>if needed</u>
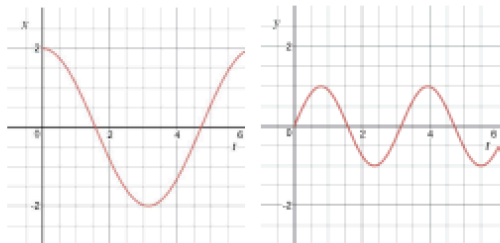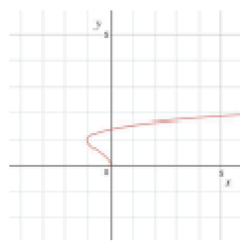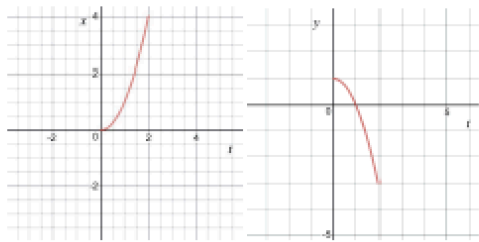

Sometimes, only a portion of the curve is specified

Example: Sketch $\begin{cases} x = 2\sin t \\ y = 3\cos t \end{cases}$ $\frac{\pi}{2} \le t \le \pi$

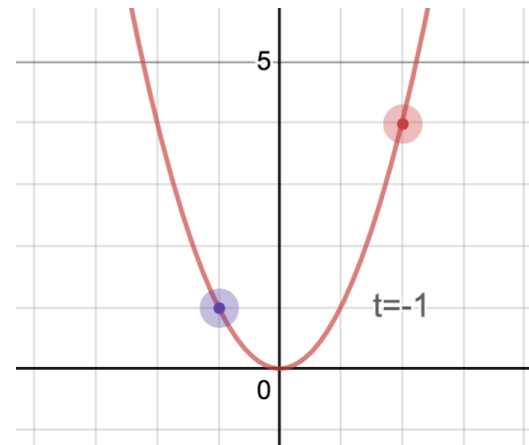More interesting animation from 5B page: https://www.desmos.com/calculator/w9ab0dxrp3

Additional Problems on Parametric Equations

Match the graphs of the parametric pair x(t) and y(t) on the left with the graph in the xy plane on the right.
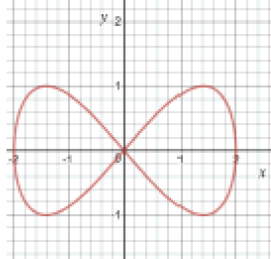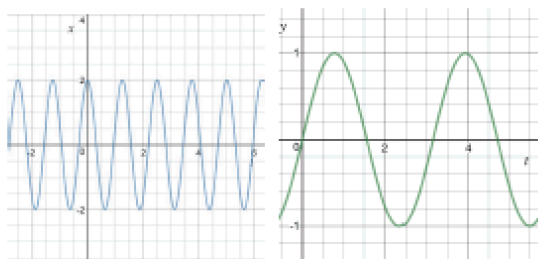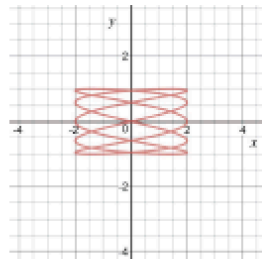
Parameterization is not unique: <u>5B page: parameterization is not unique</u>

$$\begin{cases} x = t \\ y = t^2 \end{cases} \qquad \begin{cases} x = 1 - t \\ y = (1 - t)^2 \end{cases}$$

Cross paths vs collision: 5B page <u>Collide vs Cross</u>